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A Note on the Estimation of Eddy Diffusivity and Dissipation Length in Low Winds over a Tropical Urban Terrain

ANIL KUMAR YADAV[1], SETHU RAMAN[2], and DEV DUTTA S. NIYOGI[2]

Abstract — Urban terrain poses a challenge for modeling air pollutant diffusion. In tropics, because of the dominant low wind speed environment, the importance of understanding the turbulence diffusion is even more critical, and uncertain. The objective of this study is to estimate the vertical eddy diffusivity of an urban, tropical atmosphere in low–wind speeds. Turbulence measurements at 1 Hz were made at 4-m level over an urban terrain with a roughness length of 0.78 m during winter months. Eddy diffusivity is estimated from spectral quantities of the turbulence data involving turbulent kinetic energy (E) and its dissipation rate (ε). The spectral information of the vertical velocity fluctuations is used to estimate the vertical length scale which provides information on the eddy diffusivity. In addition, the product of friction velocity and the vertical length scale has been used to non-dimensionalize the eddy diffusivity, which is shown to increase with increasing instability. Using the eddy diffusivity (K) estimates from the $E - \varepsilon$ approach, a relation is suggested for the mixing length based eddy diffusivity models of the form: $K = c_w.[2.5 - 0.5(z/L)]$, where z is the measurement height, L is the Obukhov length, and c_w has an average value close to 1 for unstable and near 0.5 for stable conditions for the urban terrains.

Key words: Air quality, atmospheric boundary layer, dissipation length, eddy diffusivity, tropics, urban terrain.

Introduction

Eddy diffusivity is an important variable for planetary boundary layer (PBL) parameterizations. It is used as a dimensional parameter in turbulence schemes. Following the Fickian diffusion, turbulent energy flux is assumed to flow down the gradient (STULL, 1988). In various environmental applications, there is a need to predict eddy diffusivity from knowledge of turbulence, for which the idea of quantifying vertical mixing from the spectral quantities is a viable approach (LEE, 1996).

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[1] Department of Applied Mathematics, Guru Jambheshwar University, Hisar-125001 Haryana, India.
[2] Department of Marine, Earth, and Atmospheric Sciences, North Carolina State University, Raleigh, NC 27695-7236, U.S.A.
Corresponding Author: Professor Sethu Raman, Department of Marine, Earth, and Atmospheric Sciences, and State Climate Office of North Carolina, North Carolina State University, Raleigh, NC 27695 - 7236, U.S.A. Email: sethu_raman@ncsu.edu

Recent research focussed towards understanding the turbulence structure of the atmospheric surface layer in low wind speeds (AGARWAL *et al.*, 1995; YADAV *et al.*, 1996). This research has significance, particularly in the tropics, because of the paucity of data and absence of such studies. The turbulence structure in the tropics can be different from that in the mid-latitudes, particularly under low wind conditions. In addition to developing an understanding of turbulence structure in the tropics, knowledge of eddy diffusivity is important for estimating pollutant dispersion in the lower atmosphere.

The observations used in this study are part of comprehensive turbulence and diffusion studies undertaken in the tropics as described in AGARWAL *et al.* (1995). Analyses of the turbulence data set, such as the variation of the turbulence intensities with atmospheric stability and wind speed, have revealed the presence of two characteristic regimes: one below, and the other above, 1 ms^{-1} wind speed. YADAV *et al.* (1996) studied the spectral characteristics of a turbulence data set with wind speeds more than 1 ms^{-1}. For this regime results showed that the inertial subrange features of the normalized power spectrum are generally consistent with Monin-Obukhov scaling. However, the well-established spectral relations (based mainly on the Kansas data) regarding the dependence of dimensionless dissipation rate on the stability parameter did not match with this tropical, urban observations.

There are various methods of parameterizing eddy diffusivity (cf., STULL, 1988). In a recent study, LEE (1996) examined the velocity and air temperature spectra and eddy diffusivity over forests. The study extended HANNA'S (1968) postulation that the vertical eddy diffusivity can be obtained from spectral information of the vertical velocity fluctuations. In addition to eddy diffusivity, dissipation length is another variable that is important in PBL and diffusion studies. Dissipation length scale is used to parameterize boundary layer turbulence in the models (cf. MELLOR and YAMADA, 1974; ANDRÈ *et al.*, 1978). The objective of this study is to examine, with the use of a data set covering low-wind urban stable and unstable conditions in the tropics, the stability dependence of dissipation length scale and eddy diffusivity using two different methods. Spectral and turbulence information of velocity fluctuations is used for the computation of eddy diffusivity values.

Data Description

Experimental details of the data used in this study can be found in AGARWAL *et al.* (1995), as well as YADAV *et al.* (1996). The experimental site is an urban terrain in New Delhi, India (28.43 N, 77.18 E) with an aerodynamic roughness length of 0.78 m (RAMAN *et al.*, 1990). The turbulent (1 Hz frequency) wind and temperature fluctuations were measured using a sonic anemometer and a fine wire thermocouple

(Campbell SWS-211/2 EK), respectively. The instruments were housed on a 30-m tower in a small field inside the Indian Institute of Technology, Delhi campus in the city. The site can be described as relatively flat, open, urban terrain with small bushes and trees at a distance greater than 500 m (YADAV *et al.*, 1996). Observations were made in stable, unstable and near-neutral atmospheric conditions. The original data set consists of 38 hourly test runs. These data were divided into two groups: one for wind speeds below 1 m s^{-1} and the other for wind speeds greater than 1 m s^{-1}, for turbulence analysis by AGARWAL *et al.* (1995). Based on the values of stability parameter and restricting the lower limit of mean wind speed to 1 m s^{-1}, 16 runs have been chosen for the present analysis. The lower limit of 1 m s^{-1} has been imposed principally due to the constraints in the instrumentation detection, and possible biases for wind speeds below 1 m s^{-1}. These 16 runs pertain to a continuous day-night period in a winter month. Figure 1 shows the mean wind speed (U) and stability parameter (z/L) variation for this period. The mean wind speed ranges between 1 and 3.5 m s^{-1}, at 4 m height. Typically unstable conditions correspond to higher winds ($\sim$ 2 to 3.5 m s^{-1}), while most stable or near–neutral conditions correspond to average wind speed between 1 and 2 m s^{-1}. Mean values of some of the relevant variables for the runs considered in this study are shown in Table 1. As seen from the data, the 16 cases present fairly diverse conditions that are representative of a tropical urban terrain.

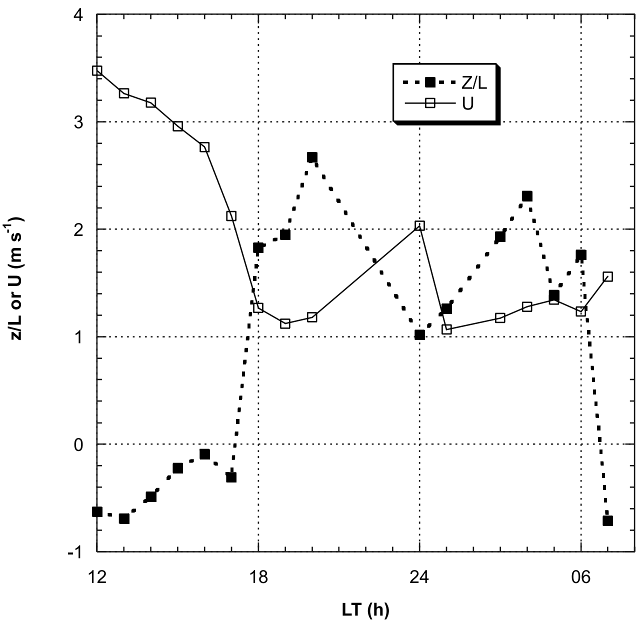

Figure 1
Variation of average wind speed (U m s^{-1}) and stability parameter (z/L) with local time.

Table 1

Some mean variables of the data set for each test run on February 14–15, 1992

Run number	Time (LST)	U (m s^{-1})	u_* (m s^{-1})	z/L	H_0 (Wm^{-2})	K (m^2 s^{-1})
1	1200	3.47	0.615	−0.63	138.42	3.337
2	1300	3.26	0.596	−0.69	147.08	3.589
3	1400	3.18	0.623	−0.49	127.54	4.559
4	1500	2.96	0.590	−0.22	47.79	3.485
5	1600	2.77	0.518	−0.09	13.87	2.682
6	1700	2.12	0.447	−0.31	29.31	1.728
7	1800	1.27	0.317	1.83	−56.42	1.194
8	1900	1.12	0.262	1.95	−31.71	0.702
9	2000	1.18	0.244	2.67	−34.01	0.794
10	0000	2.03	0.386	1.02	−44.83	1.56
11	0100	1.07	0.286	1.26	−20.25	0.79
12	0300	1.17	0.278	1.93	−24.16	0.612
13	0400	1.28	0.267	2.31	−21.86	0.669
14	0500	1.35	0.325	1.39	−23.52	0.898
15	0600	1.22	0.322	1.76	−27.58	0.668
16	0700	1.54	0.347	−0.71	10.14	1.16

Results and Discussion

Results of the spectral analysis of the 16 runs are presented in YADAV *et al.* (1996). The observed spectra were consistent with Monin-Obukhov scaling. Using derived parameters and spectral information, particularly the dissipation rate, the eddy diffusivity can be estimated.

Eddy Diffusivity

Eddy diffusivity for momentum was determined from turbulence measurements at $z = 4$ m by using two different formulations. The first is based on $E - \varepsilon$ closure and the other on HANNA'S (1968) formulation. In $E - \varepsilon$ closure, the eddy diffusivity K is parameterized as (cf. LEE, 1996),

$$K = c_\varepsilon \frac{E^2}{\varepsilon} \quad , \tag{1}$$

where c_ε is a constant (generally considered to be 0.03 to 0.04, (STULL, 1988), E is the turbulent kinetic energy (TKE), and ε is the TKE dissipation rate. As seen in Figure 1, z/L is smaller for higher wind speed. This indicates better potential for mechanical turbulence over the rough, urban terrain. The dependence of the eddy diffusivity on surface friction velocity (u_*) is shown in Figure 2. As shown in the figure, the eddy diffusivity values follow a linear relation ($r = 0.975$) which can be approximated as,

$$K = -1.85 + 9u_* \quad . \tag{2}$$

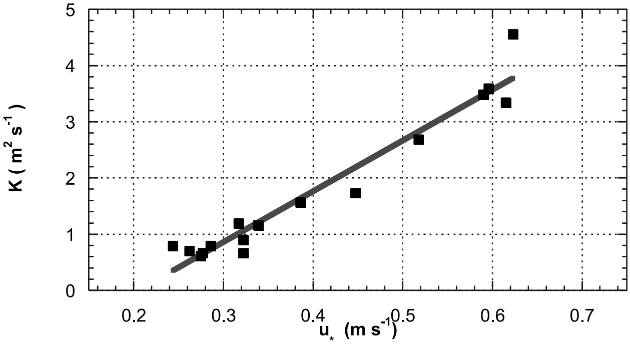

Figure 2
Variation of $E - \varepsilon$ closure based eddy diffusivity (K, m^2 s^{-1}) with friction velocity (u_*, m s^{-1}). The line shown is the best fit given by $K = -1.85 + 9\ u_*$ (r = 0.975).

The stability dependence of eddy diffusivity from $E - \varepsilon$ closure is examined further in Figure 3. The data indicate larger scatter for unstable conditions as compared to the stable regimes. Overall as expected, eddy diffusivity decreases with increasing stability. Eddy diffusivity increases rapidly for values of z/L smaller than 0 (unstable conditions). Thus in general, vertical eddy diffusivity estimation based on the $E - \varepsilon$ closure is sensitive to correct specification of the stability parameter z/L, particularly for the unstable conditions. For the unstable conditions, K ranges from about 1 m^2 s^{-1} to 5 m^2 s^{-1} while for stable conditions its value changes slightly between 0.6 to 1 m^2 s^{-1}. A quadratic best fit is also possible, of the form,

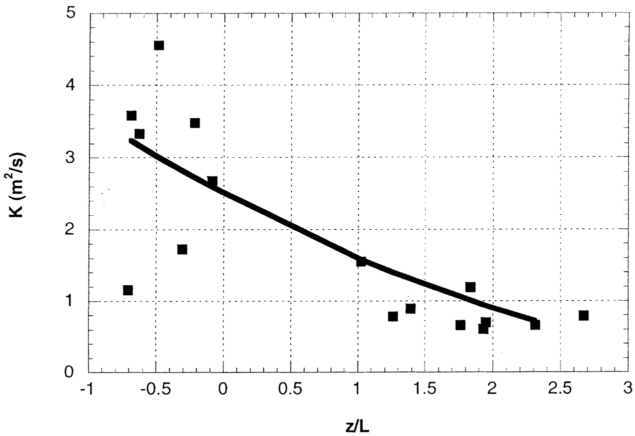

Figure 3
Variation of $E - \varepsilon$ closure based eddy diffusivity (K, m^2 s^{-1}) with the stability parameter z/L. The solid line corresponds to the quadratic best–fit of the form $A + B{\cdot}(z/L) + C{\cdot}(z/L)^2$ with A, B, and C equaling 2.5, -1.0, and 0.1, respectively.

$$K = A + B\left(\frac{z}{L}\right) + C\left(\frac{z}{L}\right)^2 \tag{3}$$

with the values of A, B, C equal to 2.5, −1.0, and 0.1, for our data set.

The other formulation for calculating eddy diffusivity is based on HANNA'S (1968) postulation. It is dependent upon the gross characteristics of the vertical velocity component of turbulent eddies. Accordingly, diffusivity K can be obtained from spectral features of the vertical velocity component, and can be expressed as,

$$K = c \cdot \frac{\sigma_w^4}{\varepsilon} \ , \tag{4}$$

where c is a proportionality constant. HANNA (1968) proposed a value of $c = 0.3$ while LEE (1996) suggested a broader range depending on the level of turbulence, averaged to a value close to 0.41. Equation (4) can also be written as,

$$K = c_w \cdot l_w \cdot \sigma_w \tag{5}$$

where,

$$l_w = \frac{\sigma_w^3}{\varepsilon} \ . \tag{6}$$

l_w is called the vertical integral scale and is a measure of the vertical size of the eddies responsible for most of the mixing (TENNEKES and LUMLEY, 1972). The variation of the constant c_w with stability has been studied by LEE (1996), using surface layer similarity functions and local similarity relations suggested by NIEUWSTADT (1984). In our study, the value of c_w for neutral conditions obtained by comparing HANNA'S (1968) formulation with $E - \varepsilon$ formulation was considerably higher than that which many investigators consider typical (0.3). We attribute this elevated value to the increased roughness and higher turbulence levels within the urban study site. LEE (1996) also found similar higher values over vegetation canopies with high surface roughness. Our c_w values range from 0.6 to 1.45 (with an average value around 1) for the unstable conditions, and from 0.37 to 0.73 (with a mean around 0.5) for the stable conditions.

Excluding the proportionality constant c_w, the product $l_w{\cdot}\sigma_w$ has been plotted for the present data against the stability parameter z/L in Figure 4. On the unstable side the scatter is relatively less, as compared to the $E - \varepsilon$ based diffusivity values shown in Figure 3. The variation for the stable conditions is nearly similar for both cases (Figs. 3 and 4). For the two parameters ($l_w{\cdot}\sigma_w$ and z/L) a linear variation (r = 0.85) can be obtained of the form,

$$l_w\sigma_w = 2.5 - 0.5\left(\frac{z}{L}\right) \ . \tag{7}$$

Combining equations (5) and (7), one can then obtain eddy diffusivity as,

$$K = c_w \cdot \left[2.5 - 0.5\left(\frac{z}{L}\right)\right] \tag{8}$$

with c_w around 1 for unstable and around 0.5 for stable conditions. Note that the value of the constant c_w is larger than that obtained by LEE (1996), since our

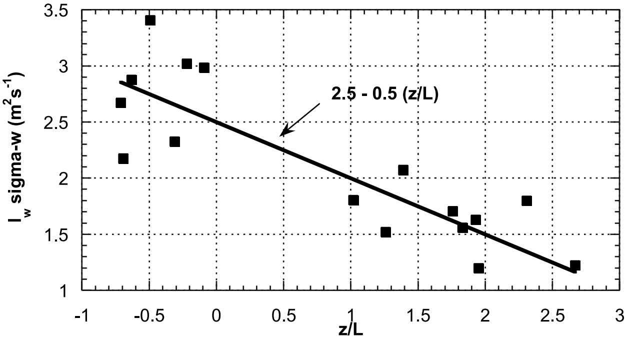

Figure 4
Variation of eddy diffusivity scale $l_w \cdot \sigma_w$ (m^2 s^{-1}) based on HANNA'S (1968) postulation with the stability parameter z/L. A best–fit of the form, $l_w \cdot \sigma_w = 2.5 - 0.5$ (z/L) is obtained.

observations span an urban terrain which has larger roughness and turbulent exchanges as compared to the forest canopy in Lee's study. However, the values are also within the ranges obtained in LEE (1996) for a different atmospheric stability.

Further, since σ_w is not routinely available, the product of friction velocity (u_*) and the vertical (l_w) may form a surrogate parameter for estimating eddy diffusivity. Figure 5 shows a scatter plot of [K/ ($u_* l_w$)] versus z/L. The trends of dimensionless K are similar to those shown in Figure 3 (for $E - \varepsilon$ case) for both stable as well as unstable conditions. Overall, the scatter is less, and is more apparent for the unstable conditions in the case of the dimensionless K. Though not shown in the figure, a linear relation can be obtained from the data, of the form:

$$\frac{K}{(u_* \cdot l_w)} = \left[1 - 0.2\left(\frac{z}{L}\right)\right] \quad . \tag{9}$$

The validity and robustness of the nondimensionalization $K/u_* l_w$ needs to be tested further with more data sets, particularly in the tropical urban domains.

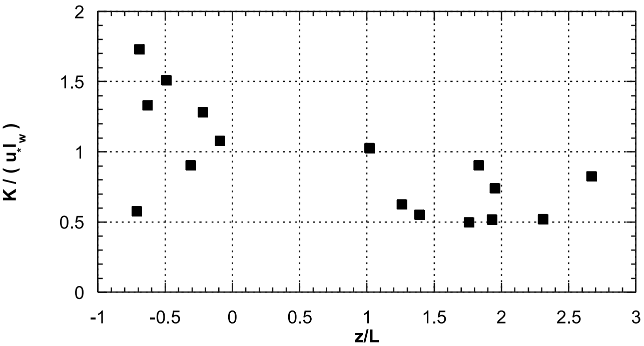

Figure 5
Variation of normalized $E - \varepsilon$ closure based eddy diffusivity $K/(u_* l_w)$ with the stability parameter z/L.

Dissipation Length

The dissipation length is an important parameter determining the level of turbulence. It provides a measure regarding the size (magnitude) of large energy-containing eddies. It varies as a function of stability and height above ground (LOUIS *et al.*, 1983), and is calculated as (TENNEKES and LUMLEY, 1972)

$$l = \frac{E^{1.5}}{\varepsilon} \ . \tag{10}$$

Figure 6 gives the time evolution of the dissipation length computed from equation (10) in terms of E and ε. When compared with the time evolution of turbulent kinetic energy and ε (not shown), a resemblance between the behaviors of these three variables is seen. At night, there appears to be some activity such as breaking of gravity waves leading to patchy turbulence. The general behavior of dissipation length with time exhibits variations principally due to the change in the stability. The computed mean values and standard deviations of the dissipation length for the urban terrain are 27.6 ± 5.6 m and 16.2 ± 3.1 m for the unstable and the stable cases, respectively. They correspond to the measurements at 4 m above the ground surface. Thus the mean value of the normalized ratio l/z is about 7 in unstable, and 4 in the stable atmospheric conditions. Over a flat and homogeneous site, LOUIS *et al.* (1983) report values of l/z around 5 for near-neutral, 10 for convective and 1 for stable conditions. Our value falls in between the near-neutral and convective values

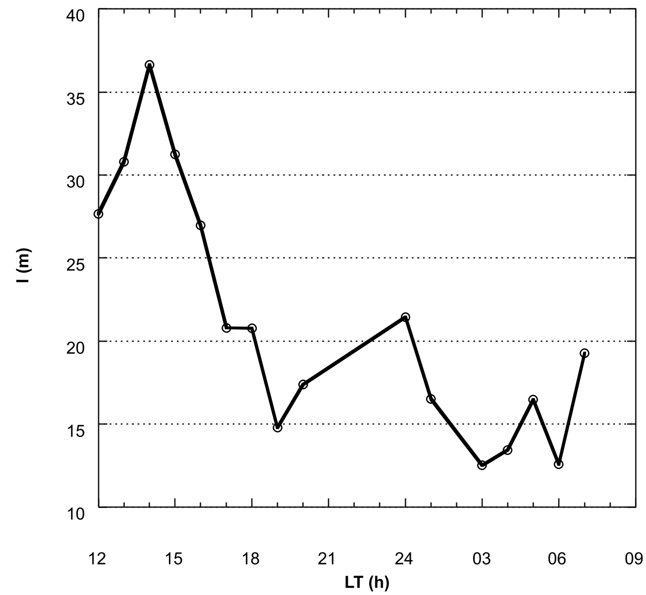

Figure 6
Diurnal variation of dissipation length (l, m) for February 14–15, 1992 for the tropical, urban terrain.

obtained by LOUIS *et al.* (1983) over flat and homogeneous terrain. On the other hand, the stable case value of l/z for the present data is higher in comparison to the corresponding value obtained by LOUIS *et al.* (1983). This could be due to roughness and other terrain characteristics, as well as wind speed conditions over the tropical, urban site in our study.

The normalized dissipation length (l/z) is plotted as a function of stability parameter z/L in Figure 7. The values show similar variations as seen for the eddy diffusivity in Figure 3.

Conclusions

Eddy diffusivity for momentum has been estimated from a data set representing low-wind, tropical, urban conditions. Two methods used for this purpose are $E - \varepsilon$ closure formulation and the HANNA'S (1968) mixing–length based postulation. The examination of stability dependence of eddy diffusivity for momentum reveals that diffusivity increases with the increase in the magnitude of the stability parameter z/L. The scatter, particularly on the unstable side, reduces when diffusivity is normalized by the product of friction velocity and vertical integral scale. A relation is suggested for eddy diffusivity estimation over urban terrain of the form: $K = c_w\,[2.5 - (z/L)]$, where c_w equals 1 for unstable and about 0.5 for stable conditions.

The dissipation length has also been estimated using the TKE and the eddy dissipation rate. Its variation with the stability indicates its sensitivity to the stability parameter z/L, particularly for the unstable conditions. The mean values of l/z in the

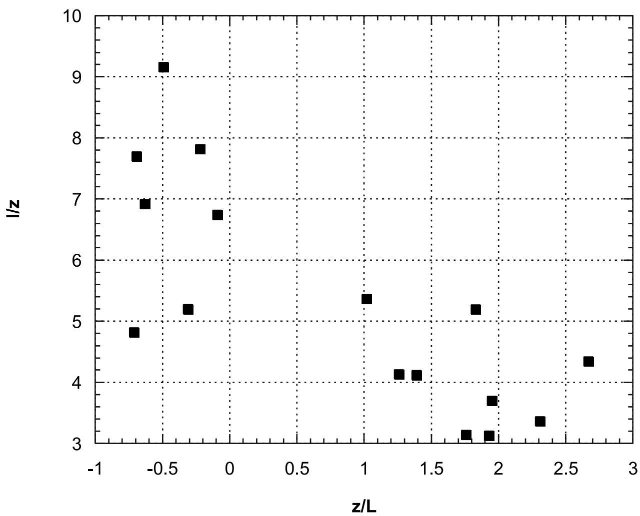

Figure 7
Variation of the normalized dissipation length (l/z) with the stability parameter z/L.

present study have been compared with those over flat and homogeneous terrain and are found to be more for the stable case and less for the unstable conditions.

Although the data are limited over a small stability range, they provide insight of the behavior of eddy diffusivity and the vertical length scale. This information can be adopted in modeling dispersion characteristics over tropical, urban regions.

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